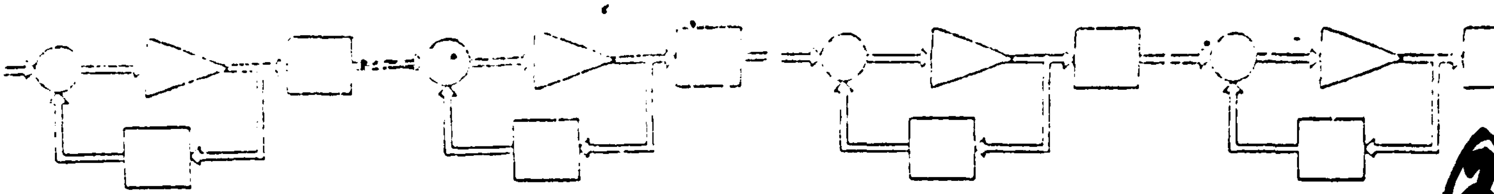

LASER WAVE ESTIMATION

Prepared For:

THE NAVAL OCEAN RESEARCH AND DEVELOPMENT ACTIVITY

NSTL Station, Mississippi 39529

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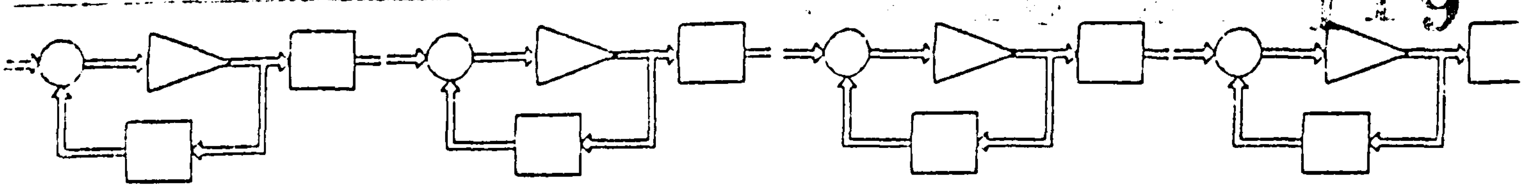

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1.0. BACKGROUND AND INTRODUCTION

1.1. Background

Airborne lasers, through measurement of the return time from the ocean surface, afford a method for the study of ocean waves and disturbances. Theoretically, given a sufficient number of measurements spread rapidly over the ocean surface beneath a low-flying aircraft, it should be possible to obtain a description of the surface at least as accurate as the laser measurement. However, the limited frequency of measurements, together with the fact that the surface is continuously changing, makes the taking of a "snapshot" of the ocean surface exceedingly difficult.

Some years ago the U. S. Navy sponsored the development of an airborne laser survey. This Navy sponsored work, References 5 to 8, generated an algorithm and a FORTRAN computer program that took time parameters, navigational data, aircraft attitude, aircraft altitude, laser beam direction, bottom and surface returns to generate a representation . the ocean floor. The underlying concept was Optimal (Kalman ilter Theory which required phrasing this estimation problem in a canonical form or model. As part of this model was a simplified representation of wave heights as Markoffian (exponentially correlated) noise. Thus, implicit in the algorithm was an estimate for wave height. However, this estimate, reflecting economies in modelling, could not reflect the full potential accuracy inherent in the laser data.

1.2. Introduction

A contract was awarded to Fagin Systems Associates, Inc., 9 September 1986, for the exploration of the potential of an airborne laser for the estimation of wave height. The approach taken was to reproduce old results, as briefly described in Section 2. It soon became apparent that the simple and economical three or four state models used in earlier work were inadequate and a full fifteen state model for a P3C aircraft was employed as described in Section 3. Section 4 describes the results obtained using this model. The results of varying wind gust standard deviation, wind gust correlation time, and the standard deviation of combined wave height and measurement error are described in Section 5. As long wave lengths, characteristic of deep ocean, were also of interest, a model variation was employed to separate wave height and measurement error, as described in Section 6. The effect of varying pulse repetition rate and beam rotation rate are discussed in Section 7. Conclusions and suggestions for possible further work are given in Section 8.

2.0. REPRODUCING OLD RESULTS

Work began by reproducing the result of Reference 5. The four state Attalt Filter Program was set up in BASIC in an IBM XT environment. A simplified measurement simulation was set up incorporating simulated measurements for a 2 degree bias error in both pitch and roll. Aside from this pitch and roll, there were no additional simulated errors. (This simple simulation is used throughout the sequel.) This program is referred to as **ATTALT2.**

The program was exercised to verify correct operation consistent with the results of Reference 6. It soon became evident that the four state model was inadequate for two reasons:-

(a) If a bias error existed in pitch, the **ATTALT2** Filter Program would come close to estimating this bias when the laser is fore or aft. However, as soon as the beam came athwartship, the estimate would drift toward zero consistent with the Markoffian model for pitch error. This would cause unacceptable errors for wave estimation. A similar argument pertains to roll. Thus, at least two additional states are necessary for pitch and roll bias error.

(b) In exercising **ATTALT2**, it was necessary to reduce the standard deviation of the pitch and roll motion to 0.2 of a degree and the vertical motion model standard deviation of vertical motion to 0.001 meter in order to achieve accuracy of estimating wave height + measurement noise to the order of 0.1 meter. Clearly, this model of aircraft motion is inconsistent with reality.

These observations led to the conclusion that a more complex model for aircraft motions was necessary. After some flirting with two state models, each for pitch, roll and vertical motion (we are up to nine states), it was decided to grab the bull by the horns and employ a full model for aircraft motion, as described in Section 3 below.

3.0. THE LOCKHEED P3C AIRCRAFT MODEL

The first foray into the realm of full aircraft motion models employed equations of motion from Reference 2, pp.171-174. There was concern that this model, for a jet aircraft, would not be representative of the P3C aircraft used for field tests. (As it turned out, partial results with this model did not differ significantly with results using the P3C model.) Consequently, Reference 3, a set of differential equations for the P3C aircraft was obtained through the good offices of Mr. John W. Clark, Jr., Aerospace Engineer at the Naval Air Development Center, Warminster, PA.

These differential equations, together with their phrasing in the canonical form of Optimal (Kalman) Filter Theory, are described in Appendix B. The flight conditions (No. 15 in Reference 3) were: 180 knots at sea level with 10 degrees flaps. No provision was made for the effects of pilot or autopilot in this model. This was a conservative assumption since, presumably, either pilot or autopilot would tend to dampen the motion of the aircraft.

The parameters given in Appendix B are referred to as the baseline model. This baseline model assumes a standard deviation of wave height plus measurement noise of 0.3 meter with each measurement having independent errors. The wind gust model assumes a standard deviation of wind gust of 0.02 radian (0.12 degree), a moderate condition. Modifications to this baseline model were made as described in the sequel.

4.0. RESULTS USING BASELINE P3C MODEL (ATTALT8)

The P3C baseline model was analyzed using the **ATTALT8** Program. Provision was made to store the fifteen standard deviations of the state vector estimates after an observation plus the fifteen estimates themselves. As RAM storage became a problem, values were stored at time intervals selected to keep the memory required reasonable.

A plotting program was developed to facilitate plotting results. The following discusses these plots for the baseline model.

4.1. Plot Of Sigma Of Wave Height + Noise Estimate For 2 Sec.

Figure 1 is a plot of the standard deviation of the estimate of wave height plus measurement noise as they are lumped together in the baseline model. The a priori standard deviation plus measurement noise is 0.3 meter. The run extended for two seconds and points were plotted after every fifth laser range measurement.

Note that the standard deviation of the estimate of wave height plus measurement noise is better than 0.1 meter---better than the accuracy of the measurement itself. Of course it is possible to know the sum of two quantities very accurately without knowing which of the two this sum represents. To obtain an approximate standard deviation of the wave estimate, the standard deviation of wave plus measurement noise must be RMS'd with the measurement noise. That is, if the standard deviation of the estimate of wave height plus measurement noise is 0.08 meter and that of the measurement error is 0.1 meter, then the

resulting standard deviation for wave height is approximately 0.128 meter. Please note that the above example implies that the limiting accuracy, as measurement error went to zero, of estimating the wave height of a 0.2 meter wave would be 0.05 meter.

The plot in Figure 1 displays considerable fundamental frequency with a 0.2 second period. Part of this can be explained by the two degree bias inserted in pitch and roll. However, removing this bias did not eliminate the fundamental component. It turns out that the mechanism for introducing this fundamental component involves a coupling between pitch error and altitude error. The pitch and altitude effects are additive when the laser points forward but subtract when pointed aft. The cancelation causes larger uncertainty when the laser points aft and minimum when pointed forward.

4.2. Plots Of The Estimates Of Pitch And Roll

Figures 2 and 3 are plots of the estimates of pitch and roll. Each Figure consists of estimates of the departure from trim condition (θ and ϕ) and an estimate of "bias" which would include equipment misalignment error plus in the case of pitch the variation of trim condition with speed and load. Note that both pitch and roll bias estimates approach 2 degrees (0.035 radians). It takes considerable time, particularly in roll, to come up with the estimate of bias. However, in each case, the sum of the estimates rapidly approaches two degrees reflecting the fact that the accuracy of the sum of the estimates is much better than that of the components.

4.3. Plot Of Sigma Of Wave Height + Noise Estimates For 10 Sec.

Figure 4 presents a plot for the same baseline conditions as for Figure 1 but extended to ten seconds. Points are plotted every 0.05 second or every 90 degrees of rotation of the beam. It seems like two different curves are plotted here. It only seems that way. The "lower curve" corresponds to the laser pointing forward which, as described above, yields a lower error. The last point on this curve has the value of 0.081 meter for the standard deviation of the estimate of wave height plus noise. In the results described below, where various parameters are varied, it is this last point at the end of ten seconds that is plotted.

5.0. THE EFFECTS OF VARYING PARAMETERS

The parameters of the wind gust model and the magnitude of the waves plus measurement noise were varied to determine their effect in the accuracy of wave height estimation.

5.1. Varying The Standard Deviation Of Wind Gusts (ATTALT7X)

A series of runs were made for different values of the standard deviations of both vertical and athwartship (αgust and βgust) wind gusts. The results of these runs are plotted in Figure 5. The results indicate that the standard deviation of the estimate of wave height plus noise is considerably improved in calmer air. However, it should be remembered that the limitation on accuracy of the estimate of wave height alone is the error in the measurement.

The aircraft can be thought of as a more or less stable platform with the laser beam swinging from side to side and fore

and aft. The less motion of the aircraft, the better the estimate of attitude, altitude and wave height plus noise. This suggests that the employment of a stable reference would significantly improve the estimate. However, practically, all is for naught unless measurement noise is decreased.

5.2. Effect Of Varying Wind Gust Correlation Time (ATTALT7Y)

The wind gust model was assumed to be exponentially correlated. Figure 6 shows the effect of varying the correlation time of this model. Note that the standard deviation of the estimate of wave height plus noise decreases above a correlation time of one second. The baseline results are seen to be conservative from these results.

These results again support the case for a stable reference to improve accuracy assuming measurement noise can be improved.

5.3. Varying Wave Height Plus Measurement Noise (ATTALT7Z)

The baseline model assumes that wave height plus measurement noise has a standard deviation of 0.3 meter. This quantity was varied with the results shown in Figure 7. Note that the error in the estimate increases with the increase of wave height plus measurement noise. This result can be explained by pointing out that in order to attain accuracy of wave height, it is necessary to accurately know aircraft height and attitude. Now the wave height, itself, is a source of noise when estimating these two quantities. Hence, increased wave height reduces the accuracy of the estimation of attitude and altitude which, in turn, reduces the accuracy of wave height plus measurement noise estimation.

6.0. SIMPLIFIED MODEL FOR LONGER WAVES (ATTALT9)

The baseline model assumes that at each laser range measurement the wave height is independent of all others. For the baseline model, the distance on the surface between successive measurements is approximately 28 meters. The assumption of independence is a valid one for shallow seas. However, for deep ocean, wave lengths can be several times the 28 meters. To explore the effect of wave length on wave height estimation, a simple exponentially correlated (in time) model for wave height was accepted.

The baseline model in Appendix B was amended as follows. The eight state in the model (formerly psi) became the measurement noise state. The fourteenth state (formerly wave height plus measurement error) became wave height alone. The standard deviation of measurement error was taken as 0.1 meter and wave height standard deviation was taken as 0.2828 so that their RMS would be equivalent to the 0.3 meter used in the baseline case.

It had been hoped that it might be easier to estimate the height of longer waves. Figure 8 dispels such illusions. The explanation for this phenomenon is similar to that for the variation of wave height. A longer wave length means fewer independent measurements for determining attitude and altitude which, in turn, impedes the accuracy of wave height estimation.

7.0. VARYING PULSE REPETITION RATE AND VARYING BEAM ROTATION RATE

Three runs were made to explore the impact on accuracy of increasing laser pulse repetition rate and beam rotation rate.

The first run was a modification of **ATTALT8** to increase the pulse repetiton rate from 400 to 800 pulses per second. This change reduced the standard deviation of the error in estimation of wave height + measurement error from 0.0822 to 0.0759 meter.

The second run was a modification of **ATTALT8** to increase beam rotation rate from five to ten revolutions per second. This change reduced the standard deviation of the estimate of wave height plus the error in measurement from 0.0822 to 0.0802 meter.

The third run incorporated both of these changes simultaneously. The pulse repetition rate was increased to 800 pulses per second and the beam rotation rate was increased to ten revolutions per second. This change reduced the standard deviation of the estimate of wave height plus measurement noise from 0.0822 to 0.0736 meter.

It should be remembered that in order to obtain the standard deviation of the estimate of wave height, these numbers must be RMS'd with the measurement error standard deviation of 0.1 meter. Thus, the standard deviation of the estimate of wave height for the above three runs is, respectively: 0.126, 0.128, and 0.124 meter. This is not much different than the baseline results of 0.129 meter. Thus, as long as measurement error remains at 0.1 meter, it is the controlling source of error and

increased pulse rate and beam rotation rate does not significantly improve the estimate of wave height.

8.0. CONCLUSIONS AND RECOMMENDATIONS

For baseline conditions, where the laser is the prime roll and pitch reference, a shallow water environment, and the wave height has a standard deviation of about 0.3 meter, it is clear that wave height can be determined to the order of the accuracy of the measurement, 0.1 meter. Reducing the measurement error will somewhat improve the accuracy of the estimate of wave height under these conditions.

In the shallow water environment, accuracy deteriorates rapidly for larger waves. Reducing the measurement error would have little effect under these conditions.

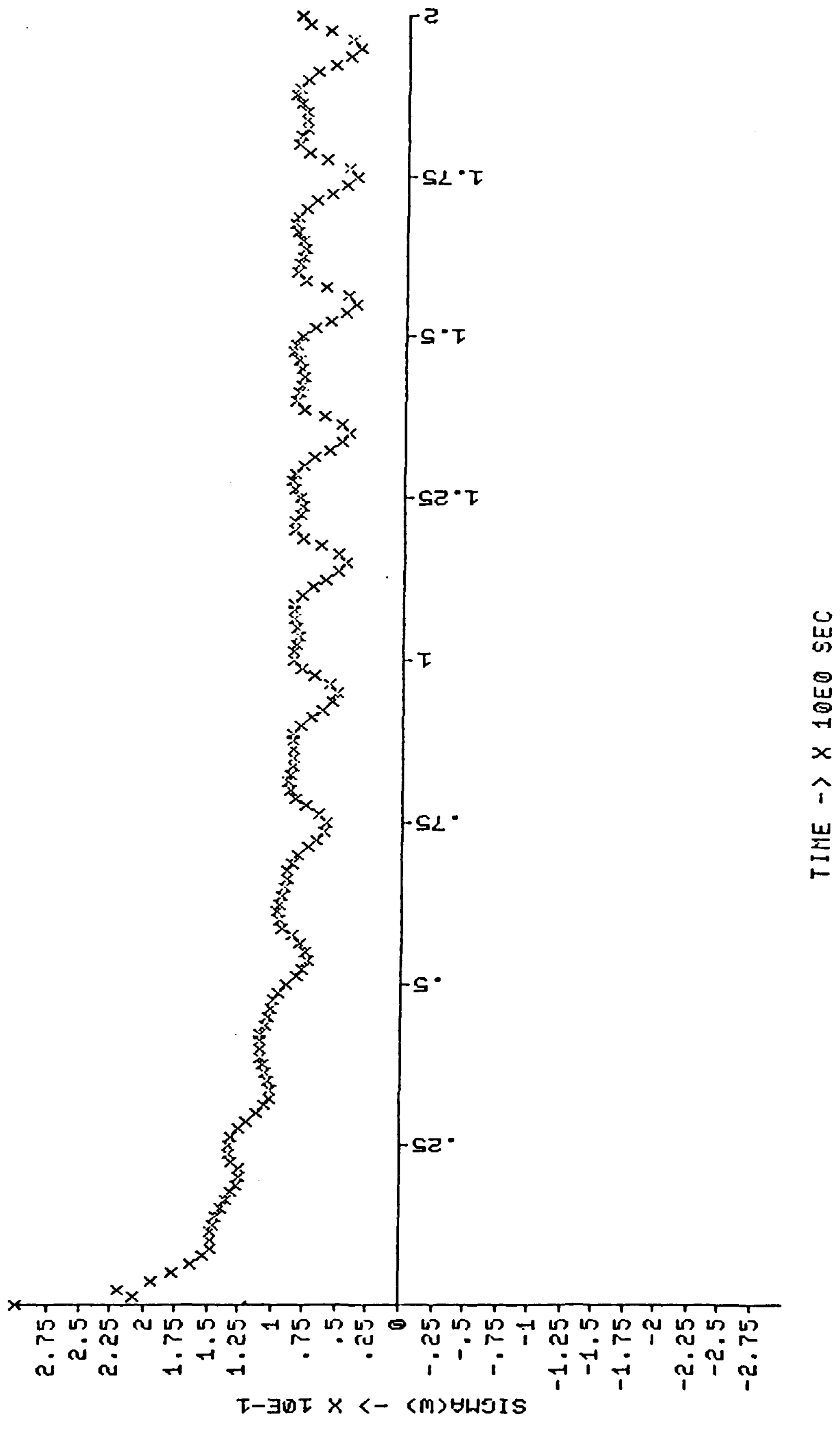
In the deep ocean environment, with longer wave lengths, the accuracy of wave height estimation decreases significantly. It should be pointed out that, in this long wave case, the errors in successive estimates would be correlated yielding a good picture of the shape of a wave but not its absolute magnitude.

If the above accuracies predicted in this report are not adequate, it might be necessary to explore the introduction of a stable reference. This might take the form of an autopilot, an Attitude and Heading Reference System, or an accurate inertial system. For the baseline condition, significant improvement would only be obtained by also reducing measurement error.

It is recommended that the several avenues for improved wave height estimation be further studied in the light of Navy

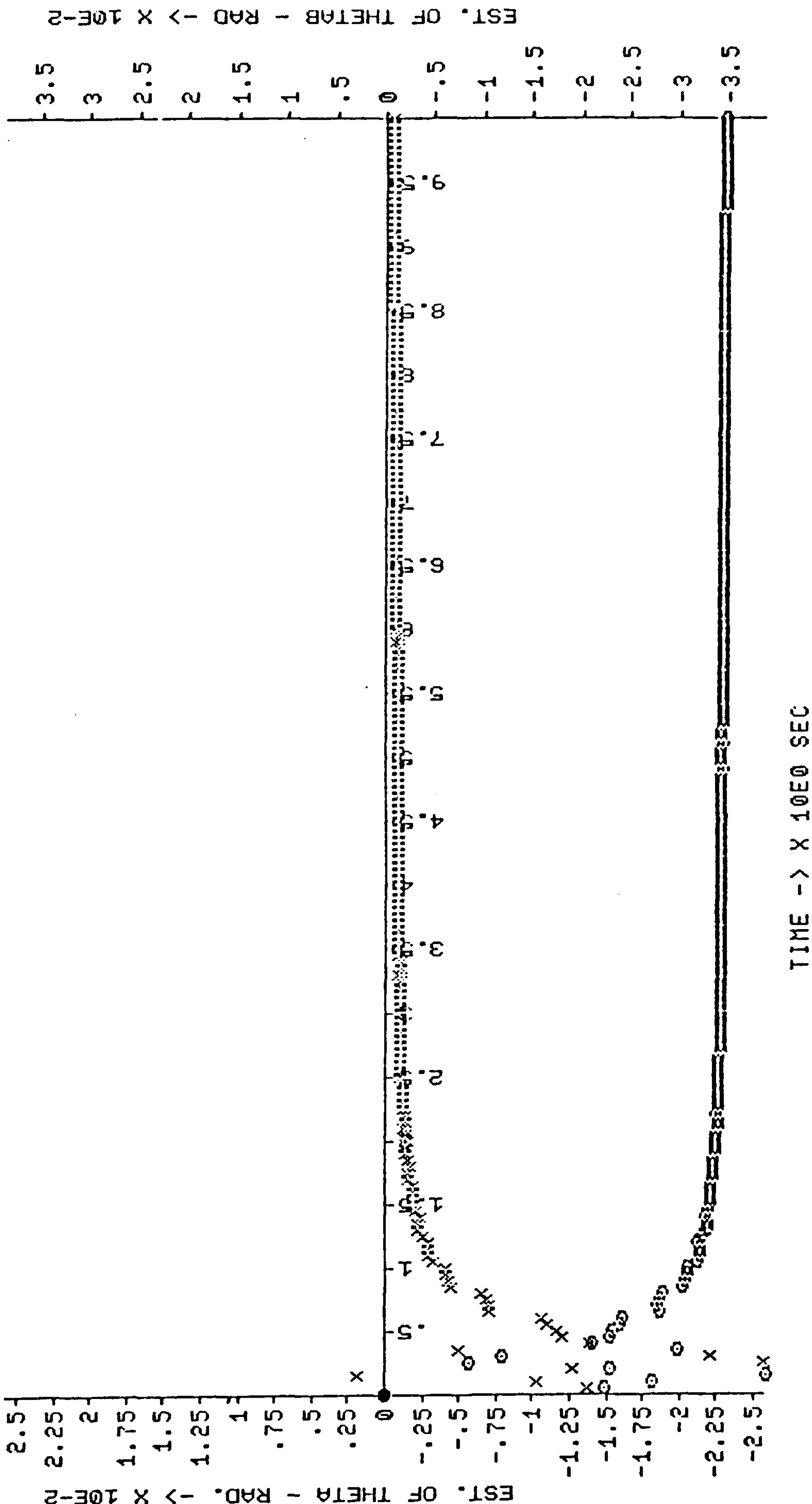
requirements. The problem is exceedingly complex and further work should be shaped by the applications.

FIGURES USED IN SECTIONS 1 TO 7



FAGIN SYSTEMS ASSOCIATES, INC. - FIGURE 1. PLOT OF S.D. OF W+MEAS ERROR ESTIMATE

FAGIN SYSTEMS ASSOCIATES, INC. - FIGURE 2. PLOTS OF ESTIMATES OF THETA & THETAB



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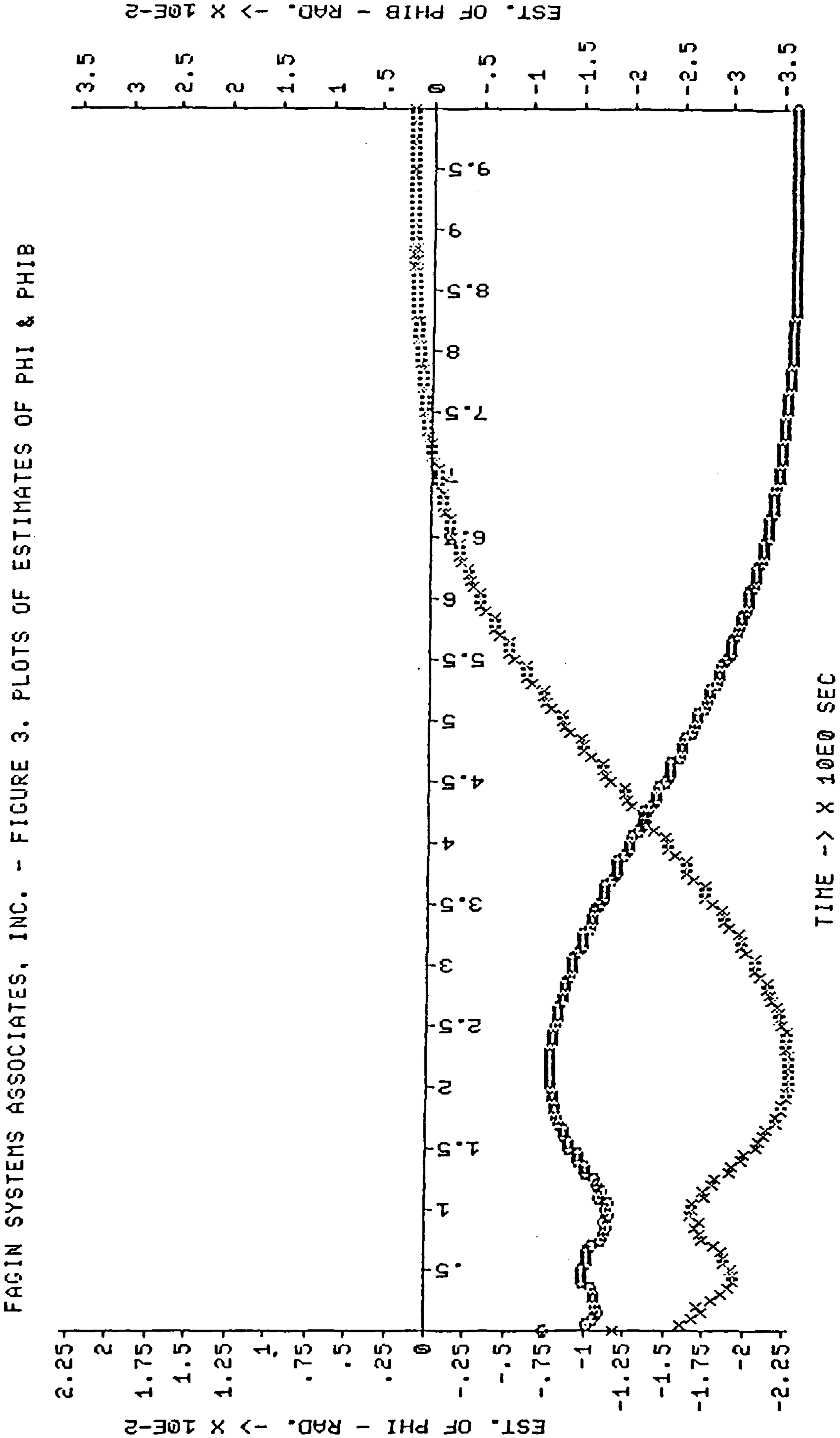
FAGIN SYSTEMS ASSOCIATES, INC. - FIGURE 3. PLOTS OF ESTIMATES OF PHI & PHIB
EST. OF PHIB - RAD. -> X 10E-2
EST. OF PHI - RAD. -> X 10E-2
TIME -> X 10E0 SEC

Laser Wave Estimation
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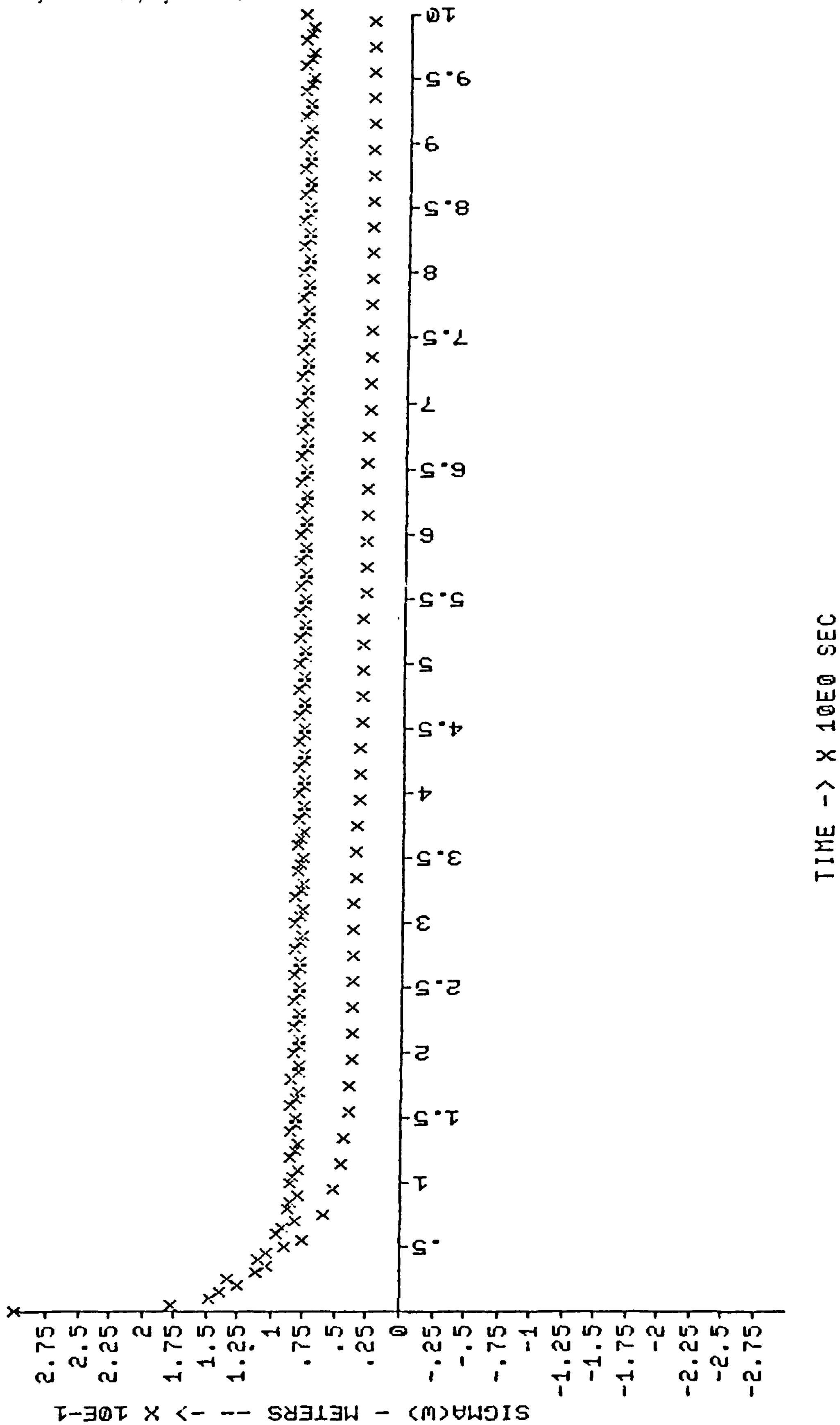
FAGIN SYSTEMS ASSOCIATES, INC. - FIGURE 4. PLOTS OF SIGMA(W+MEAS. ERR.)



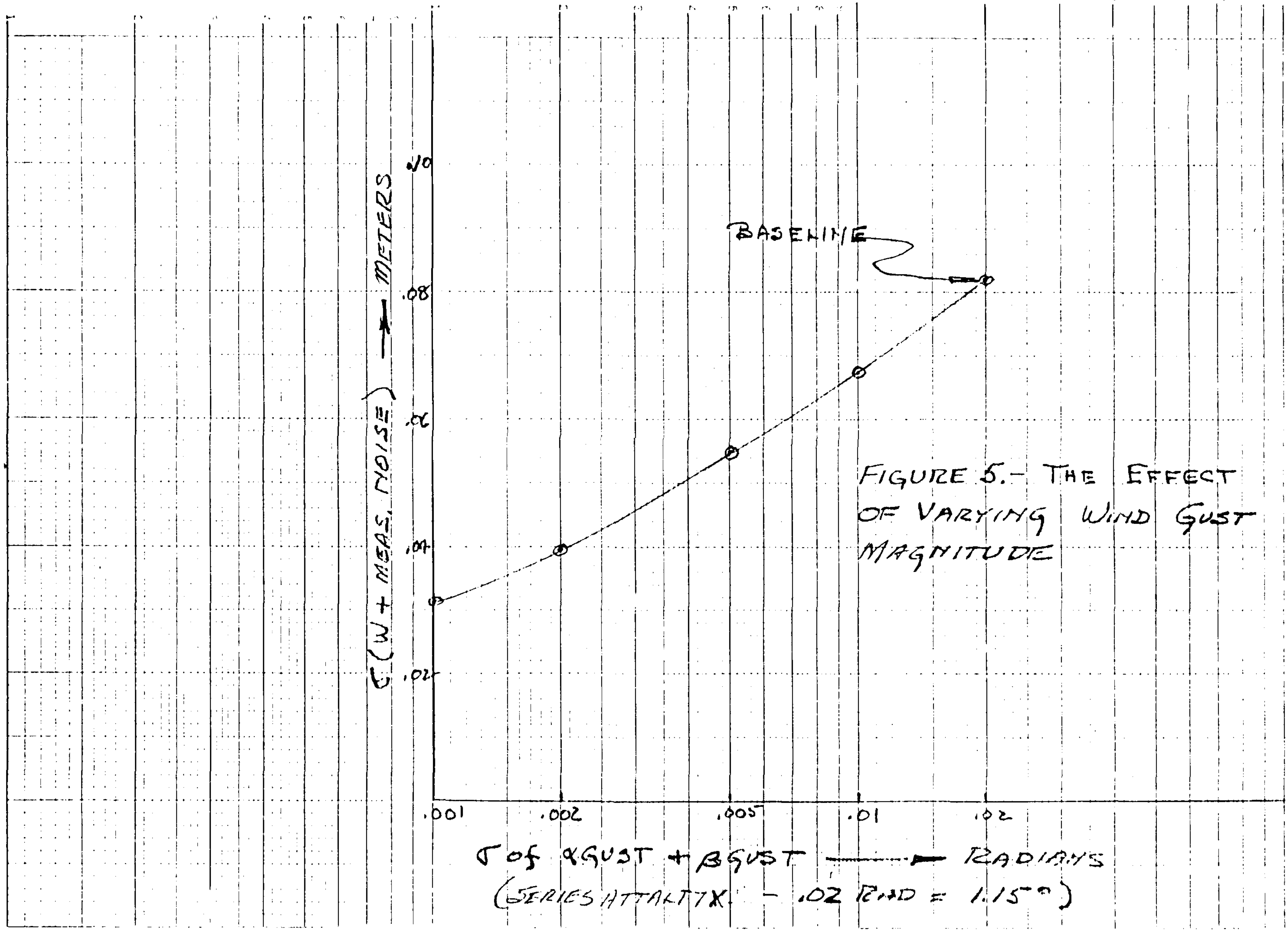


FIGURE 5.- THE EFFECT OF VARYING WIND GUST MAGNITUDE

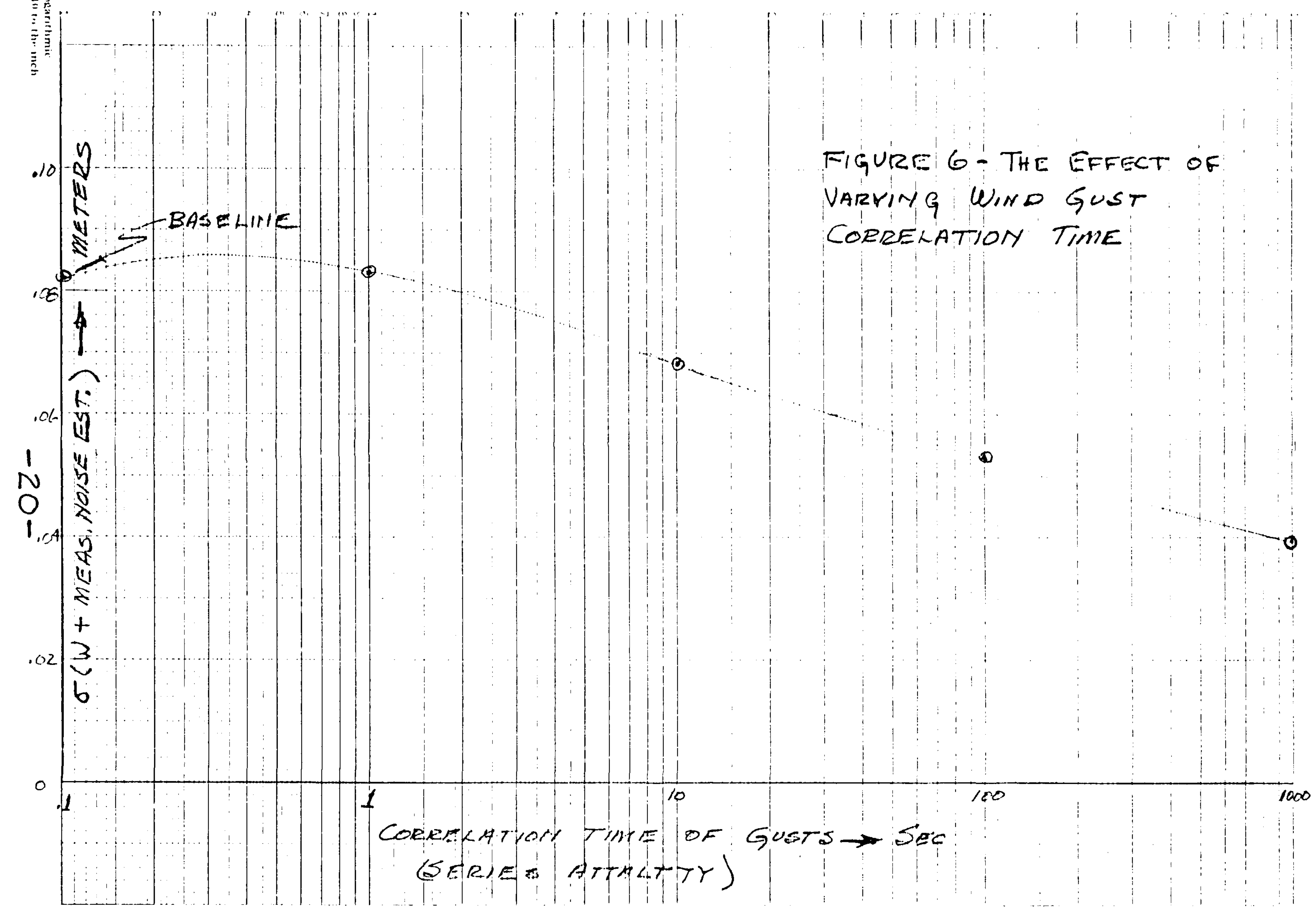


FIGURE 6 - THE EFFECT OF
VARYING WIND GUST
CORRELATION TIME
BASELINE
σ(W + MEAS. NOISE EST.) → METERS
.10
.08
.06
.04
.02
0
.1
1
10
100
1000
CORRELATION TIME OF GUSTS → SEC
(SERIES ATTALTTY)
Semi-Logarithmic
Cycles x 10 to the inch

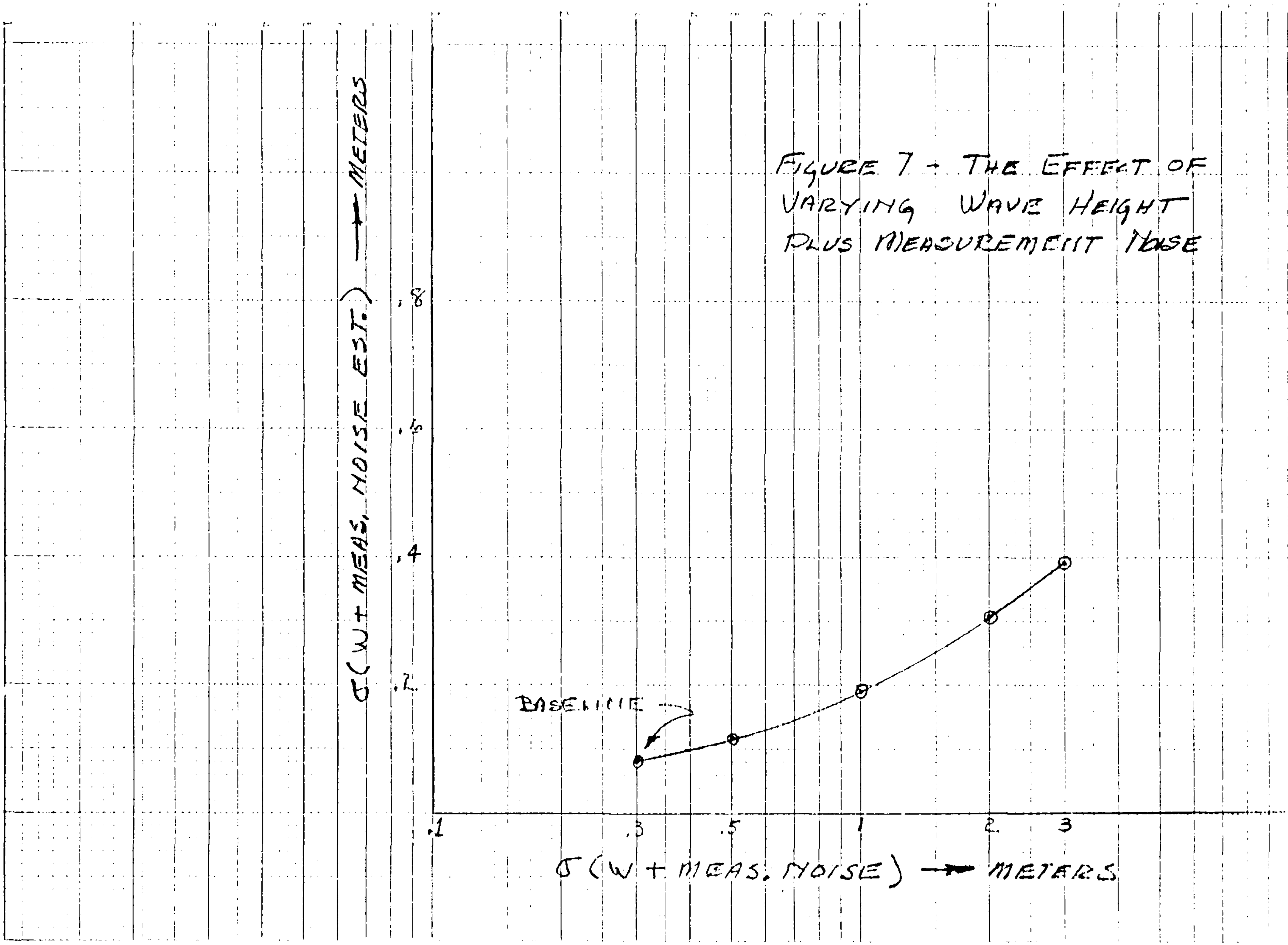


FIGURE 7 - THE EFFECT OF VARYING WAVE HEIGHT PLUS MEASUREMENT NOISE

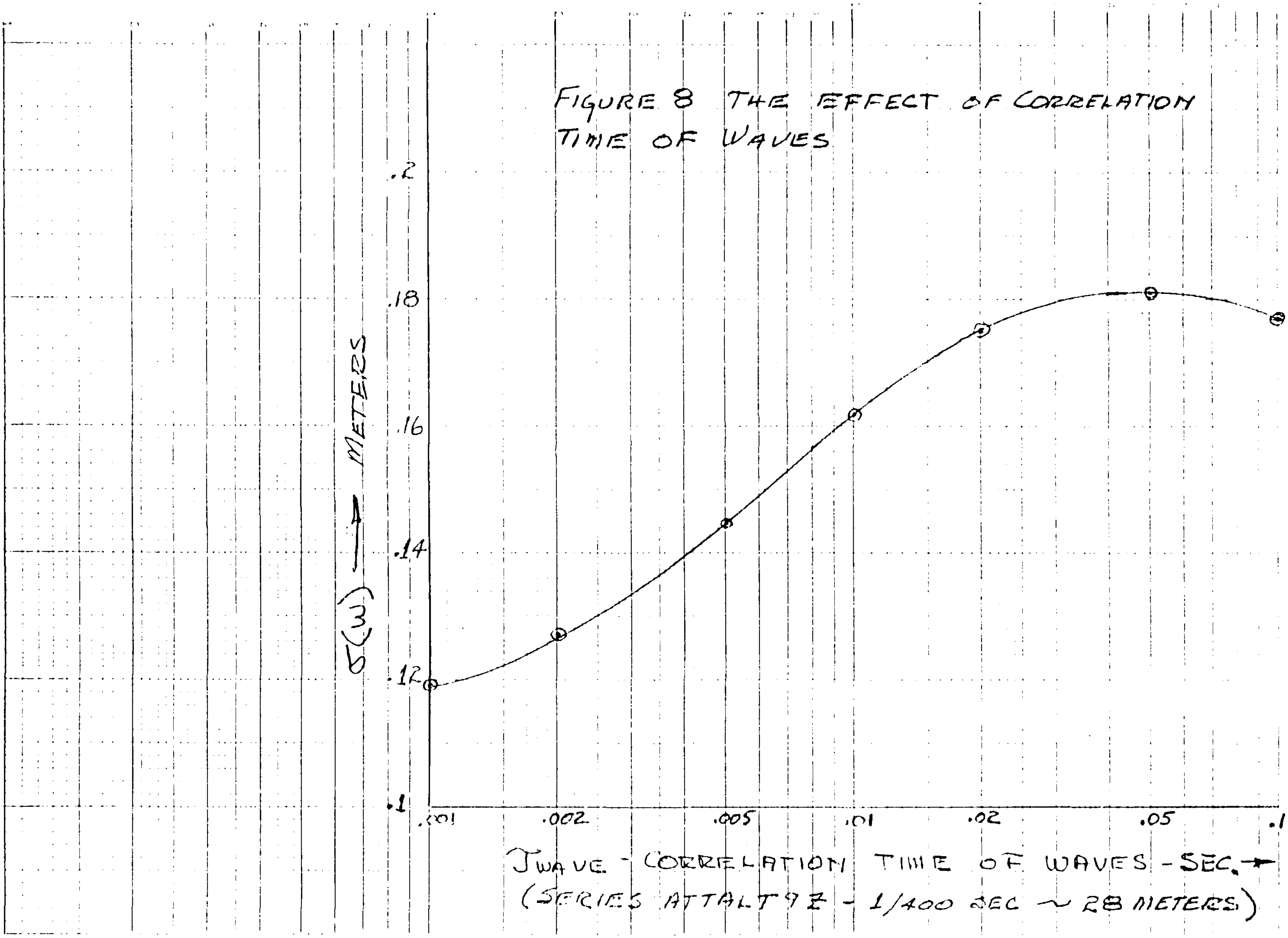


FIGURE 8 THE EFFECT OF CORRELATION TIME OF WAVES

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APPENDIX A.

THE CANONICAL MODEL

In order to employ the theory and algorithms of Optimal (Kalman) Filter Theory, it is necessary to place a problem in a special form called "The Canonical Model". A brief description of this model is given here for completeness.

In the sequel, the notation of Reference 1 is adopted. Briefly, the error model is assumed to be represented by the first order differential equations:

$$\dot{X} = A(t)X + N(t) \tag{A-1}$$

where:

- X is a "k" dimensional column "state" vector.
- $A(t)$ is a k x k "system" matrix, which may vary with time.
- $N(t)$ is a "k" dimensional column vector of white noise inputs such that

$$E\{N(t_1)N^T(t_2)\} = \pi\delta(t_1-t_2)q(t_1)$$

and

$$E\{N(t)\} = 0$$

where δ is the Dirac delta function

$q(t_1)$ is a k x k "noise" matrix.

In the modelling adopted here, the state vector X includes "noise" states such as measurement errors and error sources. Most frequently q is constant and diagonal, the diagonal elements being the single-ended power spectral density of the white noise

necessary to create the noise state of the desired variance.

At times t_i, "r" dimensional observations $Y(t_i)$ are taken defined by the equation

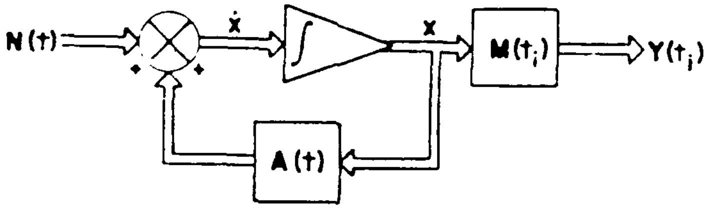
$$Y(t_i) = M(t_i)\ X\ (t_i) \qquad (A-2)$$

$M(t_i)$ is a measurement of dimension $r \times k$ where $r \le k$. $M(t_i)$ is not necessarily constant but is assumed to piecewise continuous.

Equations A-1 and A-2 can be represented by the multidimensional, Canonical Block diagram of figure A-1. In this figure, double lines indicate the flow of vector quantities, and matrix boxes multiply their vector inputs from the left.

The optimal filter or smoothing problem is to obtain a "best" estimate for the state vector "X" based on these observations and any prior knowledge of the states summed up in an initial covariance matrix P(0-).

To specify a model, it is therefor necessary to specify A, Q, P(0-), X(0-), and the measurement matrices.



$$\begin{cases} \dot{X} = A(t)\ X + N(t) \\ Y(t_i) = M(t_i)\ X\ (t_i) \end{cases}$$

Figure A-1. THE CANONICAL MODEL

APPENDIX B.

THE P3C MODEL

The differential equations for the P3C aircraft motion about trim condition in straight and level flight was obtained from Reference 7. Flight Condition 15 was chosen as most closely representing the situation of interest. Standard aerodynamic nomenclature is employed. The reader is referred to the reference for definition of terms. No effects of control surfaces were included: however, additional states were added as shown in Figure B-1, as follows:

θ_B = bias in pitch either due to the trim of the aircraft and/or misalignment.

ϕ_B = bias in roll either due to trim of the aircraft and/or misalignment.

w = wave height plus measurement error modelled as a Markoffian.

In addition, the variables α_{gust} and β_{gust} were modelled as Markoffian random variables. The resulting linear differential equation of dimension 15 is shown in Figure B-1.

The corresponding state vector "X" and the "A" matrix are shown in Figure B-2. The "q" matrix is defined by:

$$q(5,5) = 2\ \sigma^2(\alpha_{gust}) \ /\ \pi\ \tau_{gust}$$

$$q(12,12) = 2\ \sigma^2(\beta_{gust}) \ /\ \pi\ \tau_{gust}$$

$$q(14,14) = 2\,\sigma^2(w)\ /\ \pi\,\tau_w$$

All other values of the "q" matrix are zero.

The initial covariance matrix is diagonal with these having standard deviations respectively of 10, 0.1, 0.1, 0.01, 0.02, 0.1, 0.1, 0.01, 0.01, 0.01, 0.1, 0.02, 0.1, 0.3, 0.5.

A list of the remaining parameters for the baseline P3C model is given in Figure B-3. All units are Meters, Radians and Seconds.

1. $\dot{h} = U_o (\theta - \alpha)$

2. $\dot{\theta} = q$

3. $\dot{\alpha} = Z_w (\alpha + \alpha_{gust}) + (1 + Z_q / U_o) q + (Z_u / U_o) u$

4. $\dot{q} = M_w U_o (\alpha + \alpha_{gust}) + M_q q + M_u u$

5. $\dot{\alpha}_{gust} = - (1 / \tau_\alpha) \alpha_{gust} + N_{\alpha gust}$

6. $\dot{\theta}_B = 0$

7. $\dot{\phi} = p$

8. $\dot{\Psi} = r$

9. $\dot{r} = N_\beta (\beta + \beta_{gust}) + N_p p + N_r r$

10. $\dot{p} = L_\beta (\beta + \beta_{gust}) + L_p p + L_r r$

11. $\dot{\beta} = (1 / U_o) Y_\beta (\beta + \beta_{gust}) + (1 / U_o) Y_p p$
$- (1 - Y_r / U_o) r + g \phi / U_o$

12. $\dot{\beta}_{gust} = - (1 / \tau_\beta) \beta_{gust} + N_{\beta gust}$

13. $\dot{\phi}_B = 0$

14. $\dot{w} = -(1 / \tau_w) w + N_w$

15. $\dot{u} = X_u u + X_w U_o (\alpha + \alpha_{gust}) - g \theta + T_u u$

Figure B-1. P3C DIFFERENTIAL EQUATIONS

$$
X = \begin{bmatrix} h \\ \theta \\ \alpha \\ q \\ \alpha_{gust} \\ \theta_B \\ \phi \\ \psi \\ r \\ p \\ \beta \\ \beta_{gust} \\ \phi_B \\ w \\ u \end{bmatrix}
\qquad
A = \begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
0 & U_o & -U_o & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & Z_w & (1+Z_q/U_o) & Z_w & 0 \\
0 & 0 & M_w U_o & M_q & M_w U_o & 0 \\
0 & 0 & 0 & 0 & -1/\tau_\alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & -g & X_w U_o & 0 & X_w U_o & 0
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{cccccccccc}
6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\end{array}
$$

$$
\left.
\begin{array}{cccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_u/U_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_u \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & N_r & N_p & N_\beta & N_\beta & 0 & 0 & 0 \\
0 & 0 & 0 & L_r & L_p & L_\beta & L_\beta & 0 & 0 & 0 \\
0 & g/U_0 & 0 & Y_r/U_0-1 & Y_p/U_0 & Y_\beta/U_0 & Y_\beta/U_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1/T_\beta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/\tau_w & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_u+T_u
\end{array}
\right]
$$

$$
\begin{array}{ccc}
 & \begin{array}{cccccc} 10 & 11 & 12 & 13 & 14 & 15 \end{array} & \\
\begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ -1 \\ \\ \\ \\ \\ \end{array} &
\left.\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Z_u/U_o \\
0 & 0 & 0 & 0 & 0 & M_u \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
N_p & N_\beta & N_\beta & 0 & 0 & 0 \\
L_p & L_\beta & L_\beta & 0 & 0 & 0 \\
Y_p/U_o & Y_\beta/U_o & Y_\beta/U_o & 0 & 0 & 0 \\
0 & 0 & -1/T_\beta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1/\tau_w & 0 \\
0 & 0 & 0 & 0 & 0 & X_u + T_u
\end{array}\right] &
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{array}
\end{array}
$$

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1.	U_o	92.7 meters/second
3.	Z_w	-1.0530
	Z_q	-9.6066
	Z_u	-0.1904
4	M_w	-0.0103
	M_q	-1.5122
	M_u	-0.00022
5.	$\tau_{\alpha gust}$	0.1
	$\tau_{\alpha gust}$	0.02
9.	N_β	1.6394
	N_p	-0.0164
	N_r	-0.3728
10.	L_β	-1.4224
	L_p	-2.2236
	L_r	0.7342
11.	Y_β	-52.6539
	Y_p	1.0464
	Y_r	4.0938
12.	$\tau_{\beta gust}$	0.1
	$\sigma_{\beta gust}$	0.02
14.	τ_w	0.001
	σ_w	0.3
15.	X_u	-0.0152
	X_w	0.0467
	T_u	-0.0440

Units: Time = Seconds
Distance = Meters
Angles = Radians

FIGURE B-3. P3C PARAMETERS FOR BASELINE MODEL

APPENDIX C.

RELATIONSHIPS BETWEEN ERRORS IN VERTICAL, ALTITUDE AND WAVE HEIGHT

A simplified diagram, showing the relationships between altitude (H), the distance the laser beam travels to the surface (R), and the angle (A) between the beam and the vertical, is shown in Figure C-1.

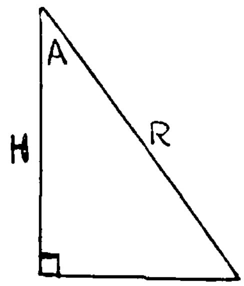

Figure C-1. SIMPLE DIAGRAM FOR LASER

It is clear that:

$$R \cos A = H$$

Taking differentials

$$dR \cos A - R \sin A \, dA = dH$$

and thus

$$dR \cos A = dH + H \tan A \, dA$$

Thus we have a relationship between the error in estimating wave height (dR cos A) and errors in altitude and the vertical. To put some numbers to this relationship assume

$$H = 500 \text{ Meters}$$

$$A = 15 / 57.3 \text{ Radians} \quad (15 \text{ degrees})$$

Then

$$dR \cos A = dH + 134 \, dA$$

It follows that if wave height error is 0.1 Meter, then dH plus 134 dA must add up to 0.1 Meter. Unless they are highly correlated, this implies that dH is the order of 0.1 Meter and dA is the order of 2.6 minutes of arc.